Engineering Notes

Optimal Reentry Time Estimation of an Upper Stage from Geostationary Transfer Orbit

M. Mutyalarao* and Ram Krishan Sharma[†] Vikram Sarabhai Space Centre, Thiruvunanthapuram, 695 022, India

DOI: 10.2514/1.44147

I. Introduction

STABLE space-debris environment is defined as an environment permitting safe space operations in the long-term future [1]. A simple way to achieve a stable space debris environment is to limit the lifetime of man-made orbiting vehicles to 25 years [2]. A spent stage that evolves along a geostationary transfer orbit (GTO) is subject to a higher collision threat potential; that is, since its path will necessarily traverse the region between geostationary orbit and low Earth orbit. Subsequently, minimization of the spent stage lifetime is one important objective in space debris mitigation efforts. The impact of natural forces such as drag and lunisolar perturbations on the lifetime of high-eccentricity orbits (less than 0.5) is the subject of earlier studies [2-7]. The effectiveness of lunisolar perturbations in reducing the lifetime of eccentric orbits with high-apogee altitudes is also the subject of previous investigations [2]. Measures such as deboosting compromise the payload capacity of the launch vehicle. Therefore, the decay of the spent stages must be carefully monitored and analyzed to facilitate enhanced planning for future missions. Specifically, minimizing the lifetime of the spent stage without jeopardizing the operational requirements.

The orbital evolution along a GTO is sensitive to the initial conditions. An accurate simulation of the process requires a good estimate of the initial state. The semimajor axis of the initial orbit is often better known than the eccentricity, because the semimajor axis is directly related to the orbital period, which is an easily measurable parameter. Therefore, the eccentricity is treated as an uncertain parameter in the present study. The ballistic coefficient $B = m/C_DA$ kg/m² is also treated as an uncertain parameter [8]. This coefficient depends on the mass of the object m, the drag coefficient C_D , and the effective area A.

The response surface methodology (RSM), or response surface approximation, is a collection of mathematical and statistical techniques developed by Box and Wilson [9]. It is useful for modeling and analysis of problems where the response is influenced by several variables [10] and the objective is to optimize the response.

The methodology is commonly employed in various fields, including agriculture and manufacturing [11]. Earlier investigations [12] predict the lifetime of a sample rocket body by combining the RSM with a genetic algorithm (GA) to estimate the eccentricity and ballistic drag coefficient. The estimation process employs the two-

line elements (TLEs) for up to 250 days. The TLE consists of two lines of formatted text. It provides the orbital elements that describe the orbit of Earth satellites at a specified epoch. It also provides the first and second time derivatives of the mean motion and the ballistic drag terms of the satellite. The osculating Keplerian elements from the TLEs are produced by removing the long- and short-periodic variations using simplified general perturbations (SGP4/SDP4) orbit theories [13]. SGP4 theory includes secular effects of J_2 , J_4 , and J_2^2 ; long-periodic effects of J_2 and J_3 ; and short-periodic effects of J_2 truncated to zeroth order in eccentricity. For an orbit with a time period less than 225 min, SGP4 theory is employed to obtain the state vector at a specified epoch. Otherwise, SDP4 theory is used to obtain the state vector. SDP4 theory includes all the terms of SGP4 plus the Earth's tesseral terms (2, 2), (3, 2), (5, 2), (4, 4), (5, 4), and the firstorder lunisolar point-mass gravity effects. The state vectors consisting of position and velocity, are obtained from the TLEs using the SDP4 theory. These state vectors are used in the Numerical Prediction of Orbital Events (NPOE) software[‡] to obtain the osculating and mean orbital elements at the initial state for the purpose of orbit propagation. The reentry bounds are computed by considering variations of up to 10% in the ballistic coefficient, the solar flux $F_{10.7}$, and geomagnetic index A_n . The actual reentry time is found to be well within the bounds.

In this Note, the reentry time of the cryogenic stage of the Indian Geostationary Satellite Launch Vehicle GSLV-F01/CS \S is carried out as an optimal estimation problem. The RSM with GA are applied to determine the optimal estimates of B and e. The investigation employs TLEs, determined 165 days before the reentry epoch, 24 November 2007, 19:30 [Greenwich Mean Time (GMT)]. The decay location is characterized by longitude of $5\degree$ N, a latitude of $2\degree$ E, and an orbital inclination of $19.3\degree$. An accurate reentry time prediction of the cryogenic stage is made seven days before its reentry. The methodology selected offers an improvement over least-squares method. The study also shows that the object must have tumbled during the last day in the orbit.

II. Methodology

A. Response Surface Methodology

As previously mentioned, RSM is a collection of mathematical and statistical techniques that is useful for modeling and analysis of problems where the response is influenced by several variables [10] called as design variables and the objective is to optimize the response. Suppose x_1 and x_2 are two independent design variables with the response y depends on them. It can be expressed as

$$y = f(x_1, x_2) + \varepsilon \tag{1}$$

where the error term ε represents any measurement error on the response, as well as other type of variations not counted in f. It is a statistical error that is assumed to distribute normally with zero mean and variance s^2 . In most RSM problems, the true response function f is unknown. To develop an approximation for f, the experimenter usually starts with a low-order polynomial. If the response can be defined by a linear function of independent variables, then the approximating function is a first-order model. If there is a curvature in the response surface, then a higher degree polynomial should be used. The approximating function with two variables is called a second-order model. Many RSM problems use either one or the

Received 4 March 2009; revision received 6 May 2010; accepted for publication 14 June 2010. Copyright © 2010 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved. Copies of this paper may be made for personal or internal use, on condition that the copier pay the \$10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code 0022-4650/10 and \$10.00 in correspondence with the CCC.

^{*}Scientist/Engineer, Applied Mathematics Division; m_mutyalarao@vssc.gov.in.

[†]Head, Applied Mathematics Division; rk_sharma@vssc.gov.in.

[‡]Data available online at http://www.cdeagle.com/html/npoe.html [retrieved 21 June 2010].

[§]Data available online at http://www.isro.org/gslv-f01/gslv-f01.aspx [retrieved 21 June 2010].

Serial no.	Epoch (UTC)	Ballistic coefficient B, kg/m ²	Osculating eccentricity e	Predicted reentry time (UTC)	% error
1	6 Oct. 2007 05:41:37	87.100	0.4059951	20 Nov. 2007 19:41	-8.064
2	9 Oct. 2007 05:46:38	87.700	0.3991590	20 Nov. 2007 22:10	-8.362
3	15 Oct. 2007 08:20:21	81.200	0.3844066	21 Nov. 2007 02:20	-9.195
4	5 Nov. 2007 01:23:16	98.800	0.2899391	23 Nov. 2007 08:03	-7.506
5	13 Nov. 2007 07:56:29	122.901	0.2233379	24 Nov. 2007 21:19	0.604
6	17 Nov. 2007 09:37:31	141.081	0.1857583	24 Nov. 2007 20:12	0.308
7	22 Nov. 2007 20:14:32	89.300	0.1036098	24 Nov. 2007 20:02	0.808

Table 1 Computed values of initial ballistic coefficient, eccentricity, and expected reentry using RSM with GA

mixture of both of these models. To obtain the approximation of polynomials, an experimental design [14] must be used to collect the response data. Once the data are collected, the method of least-squares [15] is used to estimate the parameters in the polynomials. The response surface analysis is performed by using the fitted surface. The objective of using RSM can be accomplished by the following:

- 1) Gain understanding of the topography of the response surface (local maximum and local minimum).
- 2) Use an optimization technique to estimate the design variables for which the response is optimal.

As previously mentioned, the estimate of orbital eccentricity is not as accurate as that of the semimajor axis. Furthermore, the ballistic coefficient is also uncertain since the attitude of the rocket body is not well known. Therefore, we use the two parameters initial osculating eccentricity \boldsymbol{e} and ballistic coefficient \boldsymbol{B} as the design variables to update the knowledge of the initial state, which will subsequently affect estimate of reentry time. The propagated mean apogee altitude for the specified initial osculating eccentricity and ballistic

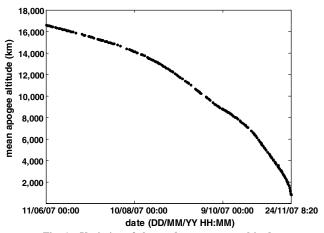


Fig. 1 Variation of observed mean apogee altitude.

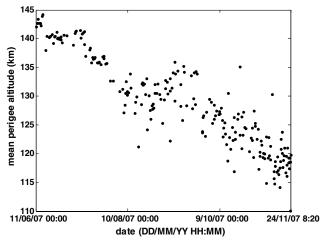


Fig. 2 Variation of observed mean perigee altitude.

Table 2 Computed values of osculating semimajor axis and eccentricity using NPOE with Earth's gravity model plus drag plus lunisolar gravity

_	Five days after 13 Nov. 2007 07:56:29 (UTC) ^a						
	Osculating semimajor axis, km	Osculating eccentricity					
$J_{2,2}$	7952.767	0.1835009576					
$J_{4,4}$	7958.693	0.1839823585					
$J_{6,6}$	7960.374	0.1841259306					
$J_{10.10}$	7960.151	0.1840969156					
$J_{18,18}$	7959.874	0.1840817827					

^aInitial osculating semimajor axis is 8360.767544 km and eccentricity is 0.223517537.

coefficient is referred to here as *mean apogee surface*. It may be noted that the variations of the mean apogee surface reflects the dynamics of the orbital motion. The present methodology is as follows:

The state vector and osculating orbital elements are obtained from the TLEs with the help of the SGP4/SDP4 theories [13]. We select three initial values of the osculating eccentricity (e_1 , e_2 , and e_3) in such a way that the predicted mean perigee at the initial epoch is within ± 3 km of the observed mean perigee altitude. Using e_2 , we select three values of the ballistic coefficients (B_1 , B_2 , and B_3) for orbit propagation by keeping the other orbital parameters unchanged, in such a way that the observed mean apogee altitudes during the interval of our study fall well within the lower and the upper bounds of the mean apogee surfaces.

- 1) With the estimates of e and B, we generate nine mean apogee surfaces. These mean apogee surfaces are used to compute the predicted mean apogee altitude at a specified epoch by a method of interpolation.
- 2) An optimization technique, GA in the present study, is employed to estimate initial e and B which minimizes the error between the observed and the predicted mean apogee altitudes.

B. Prediction of Reentry Time for GSLV-F01/CS

Using the above methodology, the estimation of the initial ballistic coefficient and the initial osculating eccentricity is made from different epochs. Seven epochs between 6 October 2007 and 22 November 2007 are chosen for carrying out the reentry time prediction study. Table 1 provides the seven epochs, estimated values of B and e, predicted reentry time, and the percentage error {i.e., percentage error is equal to [(predicted reentry time minus reference time) divided by (reference reentry time minus time of the TLE epoch)] multiplied by 100} on each of the reentry predictions. It may be noted that in this span, the mean perigee altitude varies between 114 and 127 km. To estimate B and e for the first four epochs, the orbital data up to 13 November 2007 are used. The orbital data used for the fifth, sixth, and seventh cases are from 13 November 2007 to 17 November 2007, 17 November 2007 to 22 November 2007, and 22 November 2007 to 24 November 2007, respectively. The observed mean perigee and the mean apogee altitudes of the rocket body from 11 June 2007 to 24 November 2007 are plotted in Figs. 1 and 2, respectively. For assessing the effect of the higher-fidelity gravity model of Earth, NPOE is used for five days from 13 November 2007 7:56:29 [Coordinated Universal Time (UTC)].

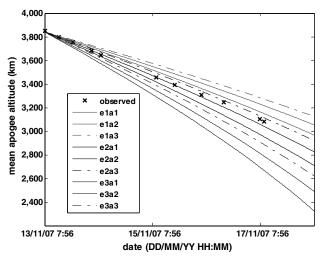


Fig. 3 Mean apogee surfaces propagated with e_1 , e_2 , and e_3 equal to 0.2231, 0.2233, and 0.2235 and B_1 , B_2 , and B_3 equal to 110, 120, and 130.

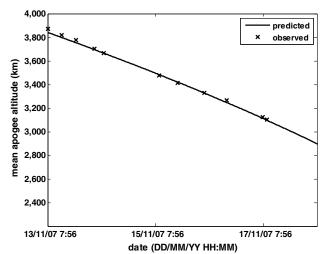


Fig. 4 Comparison of observed and predicted mean apogee altitudes.

Terms up to $J_{2,2}$, $J_{4,4}$, $J_{6,6}$, $J_{10,10}$, and $J_{18,18}$ of the Earth gravity model based on GEM10B [16], along with the atmospheric drag and lunisolar gravity effects, are considered for orbit propagation. Table 2 provides the computed osculating semimajor axis and eccentricity after five days from 13 November 2007 7:56:29 UTC. It may be noted that the terms up to $J_{6,6}$ of the Earth gravity model based on GEM10B are adequate for the present study. From Table 1, a case is chosen for showing the details of the employed methodology:

C. Case Studies

The osculating orbital elements for the TLE of 13 November 2007, 7:56:29 UTC are as follows. The semimajor axis is 8360.767544 km, eccentricity is 0.223517, inclination is 19.332897 deg, argument of

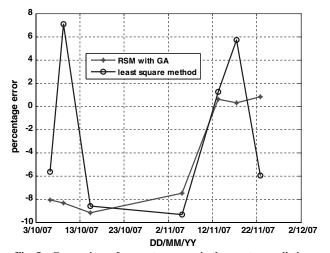


Fig. 5 Comparison of percentage error in the reentry prediction.

perigee is 76.399354 deg, right ascension of ascending node is 252.596863 deg, and true anomaly is 280.139374 deg.

To generate a set of mean apogee surfaces, three values of initial osculating eccentricity (e_1 , e_2 , and e_3 equal to 0.2231, 0.2233, and 0.2235) and three values of the ballistic coefficient $(B_1, B_2, \text{ and } B_3)$ equal to 110, 120, and 130) are selected to obtain nine grid points. Daily variations of $F_{10.7}$ and A_n are included for orbit propagation. Figure 3 shows the set of mean apogee surfaces with these initial osculating eccentricity and ballistic coefficient values. GA is applied to determine the optimal estimates of e and B. The values of e and Bso obtained are 0.2233379 and 122.90146 kg/m², respectively. Figure 4 shows the comparison between the observed and the predicted mean apogee altitudes. The predicted reentry epoch of GSLV-F01/CS, with these values of e and B and with the predicted values of $F_{10.7}$ and A_p , is 24 November 2007 21:19 UTC. It may be noted that the reentry time prediction of the GSLV-F01/CS is made seven days before its reentry. The lower and upper bounds for reentry time, obtained with the estimated extreme values of $F_{10.7}$ and A_p , are 24 November 2007 19:17 UTC and 25 November 2007 00:12 UTC, respectively.

D. Least-Squares Method

To compute the initial values of e and B, the least-squares method [15] is employed as follows:

First, the mean values of apogee and perigee altitudes, at different epochs, are determined from the available TLEs. This is accomplished through the use of the NPOE software, the resulting values referred to here as the observed values. The resulting data are used in a least-squares process to determine a polynomial fit (of degree 6) that estimates the perigee and the apogee altitudes at the initial time. These estimates are used to compute the initial values of the mean semimajor axis and the mean eccentricity.

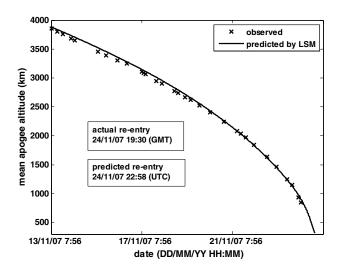
The estimated semimajor axis and eccentricity, and the other mean elements obtained from the TLEs, are used to propagate the trajectory with the NPOE software. The force model used includes terms up to $J_{6,6}$ in the Earth gravity model based on GEM10B, atmospheric drag perturbations and lunar and solar gravity effects. The MSIS90 density model [17] including the daily values of $F_{10,7}$

Table 3 Computed values of initial ballistic coefficient, eccentricity, and expected reentry using least-squares method

Serial no.	Epoch (UTC)	Ballistic coefficient B, kg/m ²	Osculating eccentricity e	Predicted reentry time (UTC)	% error
1	6 Oct. 2007 05:41:37	87.543	0.405946	22 Nov. 2007 00:16	-5.651
2	9 Oct. 2007 05:46:38	97.813	0.399060	28 Nov. 2007 02:24	7.059
3	15 Oct. 2007 08:20:21	114.751	0.384631	21 Nov. 2007 08:08	-8.584
4	5 Nov. 2007 01:23:16	79.933	0.289772	22 Nov. 2007 23:10	-9.351
5	13 Nov. 2007 07:56:29	196.800	0.223416	24 Nov. 2007 22:58	1.258
6	17 Nov. 2007 09:37:31	136.368	0.185706	25 Nov. 2007 05:42	5.734
7	22 Nov. 2007 20:14:32	103.486	0.103819	24 Nov. 2007 16:40	-5.996

Table 4 Comparison	i between	observed at	id estimated	narameters t	for GS	V-FOI/CS in	orbit
--------------------	-----------	-------------	--------------	--------------	--------	-------------	-------

TLE Epoch from-to (UTC)	Lat observed [computed], deg	Long. observed [computed], deg	Mean perigee altitude observed [computed], km	Mean apogee altitude observed [computed], km	Altitude observed [computed], km	Ballistic coefficient, kg/m ²
19 Nov. 2007 11:31:29 to	0.2911	212.0996	118.705	2522.143	1971.564	116.3
19 Nov. 2007 20:52:12	[0.2611]	[212.0760]	[117.944]	[2522.185]	[1969.025]	
19 Nov. 2007 20:52:12 to	0.3671	43.1092	119.871	2402.971	1951.341	114
20 Nov. 2007 07:57:57	[0.4841]	[43.3612]	[117.559]	[2402.787]	[1952.763]	
20 Nov. 2007 07:57:57 to	0.0073	320.03	117.938	2080.298	1878.844	114.6
21 Nov. 2007 12:54:48	[0.0935]	[320.3854]	[117.635]	[2081.305]	[1874.285]	
21 Nov. 2007 12:54:48 to	0.0645	265.93	116.304	2035.181	1854.051	116
21 Nov. 2007 16:27:55	[0.0629]	[265.9338]	[117.041]	[2033.929]	[1853.101]	
21 Nov. 2007 16:27:55 to	0.1417	185.2107	114.303	1965.582	1811.492	120.1
21 Nov. 2007 21:45:38	[0.0968]	[185.1235]	[116.554]	[1961.941]	[1811.155]	
21 Nov. 2007 21:45:38 to	0.158	51.74	119.117	1838.572	1732.531	100
22 Nov. 2007 06:30:36	[0.2496]	[51.9226]	[117.465]	[1838.152]	[1727.603]	
22 Nov. 2007 06:30:36 to	-0.292	200.31	116.342	1636.015	1586.588	95
22 Nov. 2007 20:14:50	[-0.2583]	[200.3628]	[119.153]	[1633.615]	[1587.449]	
22 Nov. 2007 20:14:50 to	0.2213	229.854	116.925	1245.685	1237.614	106.4
23 Nov. 2007 22:48:01	[-0.0865]	[228.9487]	[116.508]	[1244.069]	[1237.661]	
23 Nov. 2007 22:48:01 to	15.33	176.36	107.692	929.308	250.105	78
24 Nov. 2007 05:46:26	[15.1380]	[177.3802]	[115.254]	[930.472]	[248.997]	
24 Nov. 2007 05:46:26 to	-0.0830	8.09	116.193	840.381	840.36	98
24 Nov. 2007 08:20:21	[0.2247]	[8.9240]	[116.478]	[838.915]	[838.895]	
24 Nov. 2007 08:20:21 to	5 [14.3947]	2 [329.6263]			 [0]	63.7
reentry epoch						



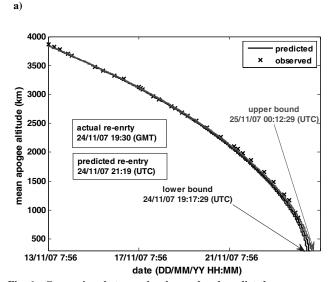


Fig. 6 Comparison between the observed and predicted mean apogee altitudes by a) least-squares method (LSM) and b) RSM with GA.

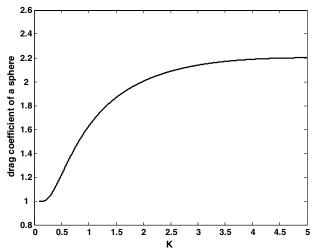


Fig. 7 Variation of drag coefficient of a sphere with respect to $K = \lambda/l$.

and A_p , is used in NPOE to compute the drag force. The mean apogee altitudes associated with various ballistic coefficients, are compared with the observed mean apogee altitudes (for 31 TLEs) for the period from 13 November 2007 to 24 November 2007. The ballistic coefficient that provides the minimum root mean square (RMS) error is chosen for computing the reentry time. It is found to be 196.8 kg/m².

The RMS error in the mean apogee altitude of the least-squares estimation is found to be 194.6397 km with the reentry time of 24 November 2007 22:58 UTC, whereas the RMS error of the RSM with GA is found to be 151.9323 km. Table 3 provides the computed values of initial ballistic coefficient, the osculating eccentricity, and predicted reentry time and percentage error using least-squares method. Figure 5 depicts the percentage errors in the prediction times for the two methods. It may be noted that toward the end of the orbital life, the RSM with GA provides better estimates of the reentry times with lesser percentage errors than the least-squares method.

Data available online at http://www.solen.info/solar [retrieved 21 June 2010].

Furthermore, the percentage errors in reentry prediction with RSM with GA reduce toward the end of life. A comparison between the observed and the predicted values of the mean apogee by the least-squares method and the RSM with GA is presented in Fig. 6.

III. Results and Discussion

Figure 7 provides the variation of the drag coefficient C_D of a sphere with respect to $K = \lambda/l$ lying between 0.1 and 5. The coefficient K provides the flow regimes experienced by the objects in orbit [8]. It depends on the mean free path λ and the characteristic length l, which is the length or a typical maximum dimension of the object. The value of K can change, depending upon the orientation of the rocket body due to its length-to-diameter ratio, which gives rise to a change in effective area and hence different ballistic coefficients. Furthermore, the ballistic coefficient B (Table 3) increases from 5 November 2007 through 17 November 2007, during which time the mean perigee altitude decreases from 120 to 117 km. On 22 November 2007, the ballistic coefficient decreased from 141 to 89. However, the estimate of reentry time is accurate, exhibiting only 0.8% error. The change in B a few days before the reentry for an object placed in high-eccentricity orbit leads to study of the orbital motion for the last few days in detail.

A detailed study is performed based on the TLEs from the last five days (11 epochs) to estimate the ballistic coefficient from one epoch to match the location of the next epoch. The results are presented in Table 4. They provide a reasonably good match between the computed and the observed values of latitude, longitude, altitude, mean perigee, and mean apogee altitudes. For the first five epochs, the ballistic coefficient varies between 114 and 120.1, then for next three epochs it is between 95 and 106.4. Since the mean perigee altitude does not vary significantly for these three epochs, m/C_D will not change appreciably; the effective area of the rocket body must have decreased near perigee, where the drag force is maximum and mainly responsible for the orbital decay. Furthermore, the variation of the ballistic coefficient suggests that the rocket body might have tumbled, causing different effective areas near each perigee passage.

IV. Conclusions

The response surface method with genetic algorithm provides optimal estimates of initial eccentricity and ballistic coefficient for high-eccentricity orbit during the end of life of the cryogenic stage of the Indian geosynchronous launch vehicle, which assists in estimating accurate reentry times from different epochs. In this regard, this method is found to be better than the least-squares method. The study shows the tumbling of the rocket body during the last day in orbit.

Acknowledgments

The authors are thankful to V. Adimurthy, Associate Director, Vikram Sarabhai Space Centre, for his kind interest and suggestions

during the course of this study. The authors are also grateful to Space-Track.Org, from where the two-line elements were obtained.

References

- [1] Klinkrad, H., Space Debris Models & Risk analysis, Springer Praxis Books Series, Springer, New York, 2006.
- [2] Klinkrad, H. (ed.), ESA Space Debris Mitigation Handbook, ESA, Darmstadt, Germany, 1999.
- [3] Sharma, R. K., Bandyopadhyay, P., and Adimurthy, V., "Consideration of Lifetime Limitation for Spent Stages in GTO," *Advances in Space Research*, Vol. 34, 2004, pp. 1227–1232. doi:10.1016/j.asr.2003.10.044
- [4] Flury, W., Janin, G., Jehn, R., and Klinkrad, H., "Space Debris in Elliptic Orbits," 18th International Symposium on Space Technology and Science, Kagoshima, Japan 1992.
- [5] King-Hele, D. G., "The Orbital Lifetime of Molniya Satellites," *Journal of the British Interplanetary Society*, Vol. 28, 1975, pp. 783–796.
- [6] King-Hele, D. G., "Lifetime Prediction for Satellite in Low Inclination Transfer Orbit," *Journal of the British Interplanetary Society*, Vol. 35, 1982, pp. 339–344.
- [7] Siebold, K. H., and Reynolds, R. C., "Lifetime Reduction of a Geosynchronous Transfer Orbit with the Help of Lunar-Solar Perturbations," *Advances in Space Research* Vol. 16, 1995, pp. 155– 161. doi:10.1016/0273-1177(95)98767-I
- [8] King-Hele, D. G., Satellite Orbits in an Atmosphere: Theory and Application, Blackie, London, 1987.
- [9] Box, G. E. P., and Wilson, K. B., "On the Experimental Attainment of Optimum Conditions," *Journal of the Royal Statistical Society Series B* (Methodological), Vol. 13, 1951, pp. 1–45.
- [10] Myers, R. H., and Montgomery, D. C., Response Surface Methodology, Wiley, New York, 1995.
- [11] Álvarez, M. J., Ilzarbe, L., Viles, E., and Tanco, M., "The Use of Genetic Algorithms in Response Surface Methodology," *Quality Technology & Quantitative Management*, Vol. 6, No. 3, 2009, pp. 295–307.
- [12] Sharma, R. K., Bandyopadhyay, P., and Adimurthy, V., "Lifetime Estimation of Upper Stages Re-Entering from GTO by Genetic Algorithm with Response Surface Approximation," International Astronautical Congress, Paper IAC-06-B6.2.11, 2006.
- [13] Hoots, F. R., and Roehrich, R. L., "Spacetrack Report No. 3: Models for Propagation of NORAD Element Set," U.S. Air Force Aerospace Defense Command, Dec. 1980.
- [14] Oehlert, G. W., Design and Analysis of Experiments: Response Surface Design, W. H. Freeman, New York 2000.
- [15] Conte, S. D., De Boor, C., *Elementary Numerical Analysis: An Algorithmic Approach*, 3rd ed., McGraw–Hill, New York, 2000.
- [16] Lerch, F. J., Putney, B. H., Wagner, C. A., and Klosko, S. M., "Goddard Earth Models for Oceanographic Applications (GEM 10B and 10C)," *Marine Geodesy*, Vol. 5, 1981, pp. 145–187. doi:10.1080/15210608109379416
- [17] Hedin, A. E., "Extension of the MSIS Thermosphere Model into the Middle and Lower Atmosphere," *Journal of Geophysical Research*, Vol. 96, 1991, pp. 1159–1172. doi:10.1029/90JA02125

B. Marchand Associate Editor